Subproblem Finite Element Method for Current and Voltage Driven Magnetic Devices

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A progressive modeling of magnetic devices is performed via a subproblem finite element method. A complete problem is split into subproblems, possibly defined on different adapted overlapping meshes. Model changes are performed for added materials and for physical or geometrical model improvements. The subproblems can be current or voltage driven. The proposed unified procedure efficiently feeds each subproblem via interface conditions, which lightens mesh-to-mesh sources transfers, and quantifies the gain given by each refinement of both local fields and current and voltage global quantities.

***Index Terms*—Circuit coupling, finite element method, model refinement, subproblems.**

I. Introduction

For efficient and accurate numerical modeling of magnetic devices, an innovative step-by-step methodology is developed. It is based on a finite element (FE) subproblem (SP) method (SPM) with magnetostatic and magnetodynamic problems solved in a sequence on different adapted meshes [1-5], from simplified up to extended or accurate models of the magnetic circuits and their windings. Each step of the SPM aims at modifying the solution obtained at previous steps, by calculating reaction fields due to either some added materials [2, 5] or some model improvements [1, 3-4]. Added materials are at the basis of the SPM for which either volume sources (VSs) or, more efficiently, to lighten mesh-to-mesh sources transfers, surface sources (SSs) can be applied [4-5]. The related methodology already offers an efficient alternative to deal with source fields [5]. An additional benefit is that the developed tools for the SSs can directly be applied to other SPs, related to physical or geometrical model improvements, e.g., from perfect to real magnetic or electric materials [3], or from simplified to detailed geometries of some components [4-5].

For all the considered changes, a general method is developed herein to express the coil impedance changes after each SP, fed not only with VSs but also SSs. The resulting SP circuit relations can be used either at the post-processing step for current driven coils or during the calculation itself for voltage driven coil or general circuit coupling, which is of importance for nonlinear problems or particular excitations. This considerably extends the applicability of the SPM. Also, an attention is given to the way multiple windings can be considered together, e.g., the primary and secondary windings of transformers. The developments are done in the frame of the magnetic vector potential formulation in both 2-D and 3-D.

II. Sequenced Magnetic Models and their Sources

A. Classical sequences of magnetic models

A first considered problem (SP 1) is the one of a coil $\Omega_{s,1}$ carrying a source current density $\boldsymbol{j}_{s,1}$. The generated magnetic flux density $\boldsymbol{b}_1$ and magnetic field $\boldsymbol{h}_1$ are to be calculated in $\Omega_1 \supset \Omega_{s,1}$, being either free space or containing a region with a simplified physical model, being either a perfect electric or magnetic region, or even a region with a small enough skin depth. Such a region $\Omega_{m,1}$, of boundary $\Gamma_{m,1}$, can thus be considered via adequate boundary conditions (BCs), either $\boldsymbol{n} \cdot \boldsymbol{b}_1|_{\Gamma_{m,1}} = 0$, $\boldsymbol{n} \times \boldsymbol{h}_1|_{\Gamma_{m,1}} = 0$, with $\boldsymbol{n}$ the exterior unit normal, or an impedance BC [3]. Such an extension of this SP 1 to various configurations is a good way to point out the generality of the proposed method.

With the magnetic vector potential $\boldsymbol{a}_1$, defined such that $\boldsymbol{b}_1 = \operatorname{curl} \boldsymbol{a}_1$ and an adequate gauge condition, the related weak formulation is

$$(\mu_1^{-1} \operatorname{curl} \boldsymbol{a}_1, \operatorname{curl} \boldsymbol{a}')_{\Omega_1} - (\boldsymbol{j}_{s,1}, \boldsymbol{a}')_{\Omega_{s,1}} + \langle \boldsymbol{n} \times \boldsymbol{h}_1, \boldsymbol{a}' \rangle_{\partial\Omega_1} = 0, \quad (1)$$

where μ_1 is the magnetic permeability and $\boldsymbol{a}'$ covers a suitable set of test functions [4-5]; $(\cdot, \cdot)_\Omega$ and $\langle \cdot, \cdot \rangle_\Gamma$ denote a volume integral in Ω and a surface integral on Γ, respectively, of the product of their field arguments.

Following SP 1 with the free space calculation, an SP 2 is defined for considering the addition of a region $\Omega_{m,2}$. The related reaction fields $\boldsymbol{b}_2$ and $\boldsymbol{h}_2$ can be calculated with a weak formulation similar to (1) but with the source current density substituted with a VS or SSs [2, 4-5]. The FE mesh of the new studied domain Ω_2 can differ from the one of Ω_1, which is a key advantage of the considered approach. With VS $\boldsymbol{h}_{s,2} = (\mu_2^{-1} - \mu_1^{-1})\, \boldsymbol{b}_1$, non-zero only in the added region $\Omega_{m,2}$ where the permeability differs from the initial one [2], the weak formulation is

$$(\mu_2^{-1} \operatorname{curl} \boldsymbol{a}_2, \operatorname{curl} \boldsymbol{a}')_{\Omega_2} + (\boldsymbol{h}_{s,2}, \operatorname{curl} \boldsymbol{a}')_{\Omega_{m,2}} = 0. \quad (2)$$

Its solution $\boldsymbol{a}_2$ in Ω_2 added to $\boldsymbol{a}_1$ in Ω_1 gives the total solution $\boldsymbol{a}$ (Fig. 1, top). With SSs of trace discontinuities types, i.e., interface conditions (ICs) SSs $[\boldsymbol{n} \times \boldsymbol{h}_2]_{\Gamma_{m,2}} = -\boldsymbol{n} \times \boldsymbol{h}_1|_{\Gamma_{m,2}}$ and $[\boldsymbol{n} \cdot \boldsymbol{b}_2]_{\Gamma_{m,2}} = -\boldsymbol{n} \cdot \boldsymbol{b}_1|_{\Gamma_{m,2}}$ (or $[\boldsymbol{n} \times \boldsymbol{a}_2]_{\Gamma_{m,2}} = -\boldsymbol{n} \times \boldsymbol{a}_1|_{\Gamma_{m,2}}$), the weak formulation is [4-5]

$$(\mu_2^{-1} \operatorname{curl} \boldsymbol{a}_2, \operatorname{curl} \boldsymbol{a}')_{\Omega_2} + \langle [\boldsymbol{n} \times \boldsymbol{h}_2]_{\Gamma_{m,2}}, \boldsymbol{a}' \rangle_{\Gamma_{m,2}} = 0, \quad (3)$$

with the total solution $\boldsymbol{a} = \boldsymbol{a}_1 + \boldsymbol{a}_2$ in $\Omega_2 \backslash \Omega_{m,2}$ and $\boldsymbol{a}_2$ in $\Omega_{m,2}$ (Fig. 1, bottom). Note that discontinuity $[\boldsymbol{n} \times \boldsymbol{a}_2]_{\Gamma_{m,2}}$ is strongly defined in the function space of $\boldsymbol{a}_2$ (at the discrete level, via a fixed discontinuous tangential component acting in a layer $\Omega_{SS,2}$ of FEs only on one side of $\Gamma_{m,2}$), whereas discontinuity $[\boldsymbol{n} \times \boldsymbol{h}_2]_{\Gamma_{m,2}}$ is weakly defined in (3) (via a volume integration from (1) in that FEs layer; the source $\boldsymbol{a}_1$ for SSs is thus only needed in a reduced region, which lightens the mesh-to-mesh projection in comparison with a VS).

Following the other form of SP 1 with a simplified model of $\Omega_{m,2}$, either $[\boldsymbol{n}\times\boldsymbol{h}_2]_{\Gamma_{m,2}}$ or $[\boldsymbol{n}\cdot\boldsymbol{b}_2]_{\Gamma_{m,2}}$, or both, are to be fixed to non-zero sources, again in formulation (3), here possibly simplified with only one trace discontinuity [3] (Fig. 2).

Formulations (2) and (3) can be extended for magnetodynamics with eddy current terms in case $\Omega_{m,2}$ is a massive conducting region, e.g., a transformer tank.

B. Circuit relations of SPs for current and voltage changes

An important feature to provide is an efficient and accurate way to update the coil circuit relation, in particular to calculate the change of the coil inductance. It is known that the magnetic flux Φ_s linked to a coil can be calculated via the integration of the total $\boldsymbol{a}$ in the coil region, i.e., $\Phi_s = (\hat{\boldsymbol{j}}_s, \boldsymbol{a})_{\Omega_{s,1}}$, with $\hat{\boldsymbol{j}}_s = \hat{\boldsymbol{j}}_{s,1}$ the current density related to a unit total current, called the wire density vector [2, 6]. With the SP approach, flux Φ_s would then be $\Phi_s = (\hat{\boldsymbol{j}}_s, \boldsymbol{a}_1+\boldsymbol{a}_2)_{\Omega_{s,1}} = \Phi_{s,1} + \Phi_{s,2} = (\hat{\boldsymbol{j}}_{s,1}, \boldsymbol{a}_1)_{\Omega_{s,1}} + (\hat{\boldsymbol{j}}_{s,1}, \boldsymbol{a}_2)_{\Omega_{s,2}}$. Nevertheless, in general, the flux change $\Phi_{s,2} = (\hat{\boldsymbol{j}}_{s,1}, \boldsymbol{a}_2)_{\Omega_s}$ cannot be accurately calculated in this way because coil $\Omega_{s,1}$ is not present any more in the mesh of Ω_2. It has to be expressed in another way.

With a VS for SP 2, the key is to write (1), for $\hat{\boldsymbol{j}}_{s,1}$ and the so-normalized solution $\hat{\boldsymbol{a}}_1$, with test function $\boldsymbol{a}'$ particularized to $\boldsymbol{a}_2$ (which explicitly renders $\Phi_{s,2}$, thus with the possibility to express it in another way), and (2) with $\boldsymbol{a}' = \hat{\boldsymbol{a}}_1$, and to subtract one equation to the other. This gives after simplification

$$\Phi_{s,2} = (\hat{\boldsymbol{j}}_{s,1}, \boldsymbol{a}_2)_{\Omega_{s,1}} = -((\mu_2^{-1}-\mu_1^{-1})\,\mathrm{curl}\,(\boldsymbol{a}_1+\boldsymbol{a}_2), \mathrm{curl}\,\hat{\boldsymbol{a}}_1)_{\Omega_{m,2}}. \quad (4)$$

Such an expression only asks for an integration in the added region $\Omega_{m,2}$ of known quantities, i.e., the projection of $\boldsymbol{a}_1$ in its mesh and solution $\boldsymbol{a}_2$ by nature in the same mesh, which is a great advantage.

With SSs for SP 2, the key is to write (1), again for $\hat{\boldsymbol{j}}_{s,1}$ and $\hat{\boldsymbol{a}}_1$, with $\boldsymbol{a}' = \boldsymbol{a}_2$, and (3) with $\boldsymbol{a}' = \hat{\boldsymbol{a}}_1$, this time with $\hat{\boldsymbol{a}}_1$ non-zero only in $\Omega_1 \backslash \Omega_{m,1}$, which thus reduces the integration domain. Then, by subtracting one equation to the other, and after suitable treatments of the surface integration terms, one obtains

$$\Phi_{s,2} = (\mu_1^{-1}\mathrm{curl}\,\boldsymbol{a}_{2,s}, \mathrm{curl}\,\hat{\boldsymbol{a}}_1)_{\Omega_{SS,2}} - (\mu_2^{-1}\mathrm{curl}\,\boldsymbol{a}_2, \mathrm{curl}\,\hat{\boldsymbol{a}}_{1,s})_{\Omega_{SS,2}}, \quad (5)$$

where $\Omega_{SS,2}$ is still limited to one layer of FEs on one side of $\Gamma_{m,2}$, and $\boldsymbol{a}_{1,s}$ and $\boldsymbol{a}_{2,s}$ are the tangential traces of $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$ on $\Gamma_{m,2}$. This is a remarkable result of great utility for efficient calculations, allowing efficient SSs to be used for complex coil excitations. In case of multiple coils, (4) and (5) can independently be applied to each of them, for each associated $\hat{\boldsymbol{a}}_1$.

Expressions (4) and (5) can be used at the post-processing level in case of a current excitation to calculate the resulting coil voltage change $V_{s,2} = -\partial_t \Phi_{s,2}$ (to be added to $V_{s,1} = -\partial_t \Phi_{s,1}$ to give the total voltage) after each change. A fixed current $I_{s,1}$ in coil $\Omega_{s,1}$ asks for a zero current $I_{s,2}$ for the correction SP 2, thus with no need to represent the coil anymore.

For a voltage excitation ($V_{s,1}$ fixed), (4) and (5) have to be involved as global equations with the additional need to let the total coil current $I_{s,1} + I_{s,2}$ unknown in SP 1, the only SP involving the source coil in its mesh. The VS and SSs have thus to be modulated by this total current in (2) and (3), respectively, with a zero voltage change $V_{s,2}$, as it will be shown in detail in the extended paper.

III. Applications and Validations

The developed method for the progressive FE modeling of magnetic devices will be applied to inductors and transformers, with illustrations and validations of the proposed steps. The main elements that will be pointed out are, for each SP: the local studied domain and its proper mesh, the circuit relations relating currents and voltages, the VSs and IC-SSs received from previous SPs with efficient mesh-to-mesh projections in reduced regions, the expected accuracy on both fields and global quantities and the possibilities to improve it via other SPs.

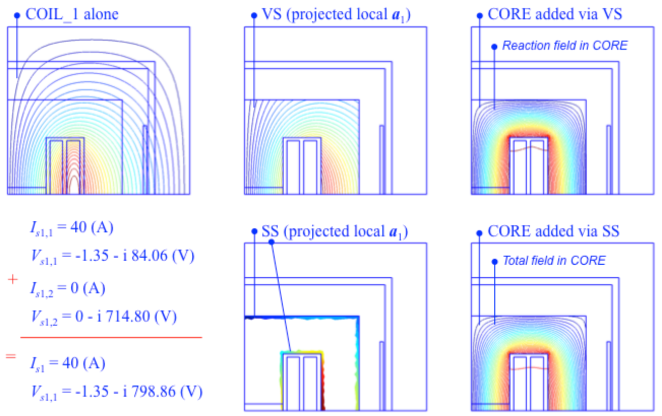

Fig. 1. Source field of a coil alone (COIL_1) acting as a VS or SSs in an added core (CORE). For a fixed current, the obtained total coil voltage $V_{s1,1} + V_{s1,2}$ is checked to be equal to the one of the complete problem with a good accuracy.

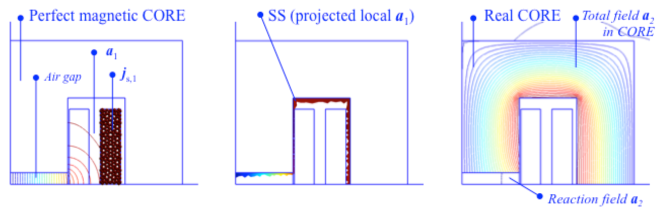

Fig. 2. Change from a perfect magnetic core with air gap to a real core via an SS (currents and voltages: $I_{s1,1} = 40$ (A), $V_{s1,1} = -11.54 - \mathrm{i}\,11257$ (V), $I_{s1,2} = 0$ (A), $V_{s1,2} = \mathrm{i}\,709.5$ (V), pointing out the importance of the correction).

References

[1] P. Dular, R. V. Sabariego, M. V. Ferreira da Luz, P. Kuo-Peng, L. Krähenbühl, "Perturbation finite element method for magnetic model refinement of air gaps and leakage fluxes," *IEEE Trans. Magn.*, vol. 45, no. 3, pp. 1400–1403, 2009.

[2] P. Dular, L. Krähenbühl, R.V. Sabariego, M. V. Ferreira da Luz, P. Kuo-Peng and C. Geuzaine. "A finite element subproblem method for position change conductor systems", *IEEE Trans. Magn.*, vol. 48, no. 2, pp. 403-406, 2012.

[3] P. Dular, V. Péron, R. Perrussel, L. Krähenbühl, C. Geuzaine. "Perfect conductor and impedance boundary condition corrections via a finite element subproblem method," *IEEE Trans. Magn.*, vol. 50, no. 2, paper 7000504, 4 pp., 2014.

[4] P. Dular, V. Péron, L. Krähenbühl, C. Geuzaine. "Subproblem finite element refinement of inductors from wire to static and dynamic volume models," *IEEE Trans. Magn.*, vol. 51, no. 3, paper 7402704, 2015.

[5] P. Dular, V. Péron, L. Krähenbühl, C. Geuzaine. "Progressive current source models in magnetic vector potential finite-element formulations," *IEEE Trans. Magn.*, vol. 52, no. 3, paper 7402804, 2016.

[6] C. Golovanov, Y. Maréchal, G. Meunier, "3D edge element based formulation coupled to electric circuits," *IEEE Trans. Magn.*, vol. 34, no. 5, pp. 3162–3165, 1998.